Random perturbations of predominantly hyperbolic maps

Alex Blumenthal

It is well known that uniformly expanding, Anosov, and Axiom A dynamical systems possess chaotic asymptotic behavior (e.g., positive Lypaunov exponent, strange attractor / SRB measure, decay of correlations). However, away from these classical settings, the verification of asymptotic chaotic behavior is notoriously difficult, even for dynamical systems with strong hyperbolic behavior on a large but noninvariant subset of phase space (what we call predominantly hyperbolic).

I will discuss in this talk some results demonstrating that the presence of possibly quite small IID random perturbations dramatically simplifies the verification of chaotic properties for maps which are predominantly hyperbolic. Models discussed will include multimodal maps of the circle (joint with Yun Yang) as well as the Chirikov standard map and 2D dissipative maps with a Henon flavor (joint with Jinxin Xue and Lai-Sang Young).

Approximation of rate functions for large deviations with application to bifurcations of random dynamical systems

Anna Maria Cherubini

We consider random maps with bounded noise and develop a numerical algorithm to approximate rate functions for large deviation principles. The algorithm is based on a Galerkin approximation of the transfer operator. We apply this to bifurcations of random dynamical systems. This work is in collaboration with G. Froyland and M. Rasmussen

Conditioned Lyapunov exponents for random dynamical systems

Maximilian Engel

We introduce the notion of Lyapunov exponents for random dynamical systems, conditioned to trajectories that stay within a bounded domain for asymptotically long times. This is motivated by the desire to characterize local dynamical properties in the presence of unbounded noise (when almost all trajectories are unbounded). The theory of conditioned Lyapunov exponents of stochastic differential equations builds on the stochastic analysis of quasi-stationary distributions for killed processes and associated quasi-ergodic distributions. We show that conditioned Lyapunov exponents describe the local stability behaviour of trajectories that remain within a bounded domain and in particular that negative conditioned Lyapunov exponents imply local synchronisation. Furthermore, a conditioned dichotomy spectrum is introduced and its main characteristics are established.

Correlation dimension and phase space contraction via extreme value theory

Davide Faranda

We show how to obtain theoretical and numerical estimates of correlation dimension and phase space contraction by using the extreme value theory. The maxima of suitable observables sampled along the trajectory of a chaotic dynamical system converge asymptotically to classical extreme value laws where: (i) the inverse of the scale parameter gives the correlation dimension and (ii) the extremal index is associated with the rate of phase space contraction for backward iteration, which in dimension 1 and 2, is closely related to the positive Lyapunov exponent and in higher dimensions is related to the metric entropy. We call it the Dynamical Extremal Index. Numerical estimates are straightforward to obtain as they imply just a simple fit to a univariate distribution. Numerical tests range from low dimensional maps, to generalized Henon maps and climate data. The estimates of the indicators are particularly robust even with relatively short time series.

Joint work with Sandro Vaienti.

Ergodicity on Sublinear Expectation and Capacity Spaces

Chunrong Feng

In my talk, I will first talk an ergodic theory of an expectation-preserving map on a sublinear expectation space. We also study the ergodicity of invariant sublinear expectation of sublinear Markovian semigroup. As an example we show that G-Brownian motion on the unit circle has an invariant expectation and is ergodic. In the last part of my talk, I will discuss the ergodic theory of Invariant capacity.

This is a joint work with Huaizhong Zhao.

A dynamical counter example for the usual interpretation of the Extremal Index

Jorge Freitas

We show how to build dynamically generated stochastic processes with an Extremal Index that does not coincide with the reciprocal of the mean of the limiting cluster size distribution. The mechanism used to build such counter examples is based on considering observable functions maximised at least two points of the phase space, where one of them is an indifferent periodic point and another one is either a repelling periodic point or a non periodic point. This enables to mix the behaviour of an Extremal Index equal to 0 with that of an Extremal Index larger than 0. Using bi-dimensional point processes we explain how mass escapes in order to destroy the usual relation.

Perturbations of dynamical systems with additive noise, noise induced order and linear response

Stefano Galatolo

Dynamical systems perturbed by noise appear naturally as models of physical systems. In several interesting cases it can be approached rigorously by computational methods. As a nontrivial example of this, we show a computer aided proof to rigorously show the existence of noise induced order in the model of chaotic chemical reactions where it was first discovered numerically by Matsumoto and Tsuda in 1983. We show that in this random dynamical system the increase of noise causes the Lyapunov exponent to decrease from positive to negative, stabilizing the system. The method is based on a certified approximation of the stationary measure in the L1 norm. This is done by an efficient algorithm which is general enough to be adapted to any dynamical system with additive noise on the interval. Time permitting we will also talk about linear response of such systems when the deterministic part of the system is perturbed deterministically.

On-off intermittency and chaotic walks

Ale Jan Homburg

We discuss the appearance of on-off intermittency in a context of skew product systems. One case we look at are iterated functions systems of maps with a common fixed point. This corresponds to skew product systems forced by a full shift. On-off intermittency occurs for vanishing Lyapunov exponents at the common fixed point. A main ingredient in the study is by a description in terms of random walks. We also consider a class of skew product maps of interval diffeomorphisms over the doubling map, from the same perspective. A main ingredient here is by a description in terms of chaotic walks: random walks driven by the doubling map.

Dynamics of Stochastic Reaction-Diffusion PDEs

Christian Kuehn

In this talk, I am going to provide a survey of some recent progress on dynamics of certain (still relatively elementary) classes of reaction-diffusion SPDEs. In particular, I shall describe how to capture local fluctuations near deterministically stationary solutions from an analytical [2,6] and a numerical standpoint [3,5,7]. Then I shall provide a brief outlook towards the case of travelling waves [1] and also explain, why for many physically more realistic models, even the basic theory of what a "solution" should be is still under very active development and discussion [4,8]. The talk is based upon a series of works with several co-authors [1-8].

(1) Warning signs for wave speed transitions of noisy Fisher-KPP invasion fronts, C. Kuehn, Theoretical Ecology, Vol. 6, No. 3, pp. 295-308, 2013.
(2) Early-warning signs for pattern formation in stochastic partial differential equations, K. Gowda and C. Kuehn, Communications in Nonlinear Science and Numerical Simulation, Vol. 22, No. 1, pp. 55-69, 2015.
(3) Numerical continuation and SPDE stability for the 2D cubic-quintic Allen-Cahn equation, C. Kuehn, SIAM/ASA Journal on Uncertainty Quantification, Vol. 3, No. 1, pp. 762-789, 2015.
(4) Regularity structures and renormalisation of FitzHugh-Nagumo SPDEs in three space dimensions, N. Berglund and C. Kuehn, Electronic Journal of Probability, Vol. 21, No. 18, pp. 1-48, 2016.
(5) Continuation of probability density functions using a generalized Lyapunov approach, S. Baars, et al., Journal of Computational Physics, Vol. 336, No. 1, pp. 627643, 2017.
(6) Scaling laws and warning signs for bifurcations of SPDEs, C. Kuehn and F. Romano. European Journal of Applied Mathematics, accepted / to appear, 2018.
(7) Combined Error Estimates for Local Fluctuations of SPDEs, C. Kuehn and P. Kuerschner, arXiv:1611.04629, 2016.
(8) Pathwise mild solutions for quasilinear stochastic partial differential equations, C. Kuehn and A. Neamtu, arXiv:1802.10016, 2018.

Conservation laws and the truncated shallow water equation.

Xue-Mei Li

In a recent paper we studied perturbation to conservation laws of stochastic systems. We discuss some examples, one of which is Lorenz equations, these are system of ODEs obtained by Lorenz by truncating the shallow water equation in a rotationary frame. They may develop turbulence and hence widely used and studied. We describe a different considerations on this old problem.

Topological conjugacy of iterated random orientation-preserving homeomorphisms of the circle

Julian Newman

Suppose we have a parameter-dependent orientation-preserving circle homeomorphism, together with a probability measure ν on the parameter space. This naturally generates a "random dynamical system" on the circle, where at each time step a parameter is randomly chosen with distribution ν (independently of all previous time steps) and the associated homeomorphism is applied. Given two parameter-dependent orientation-preserving homeomorphisms defined over the same parameter space (with the same measure), one can define a notion of "topological conjugacy" between the random dynamical systems that they generate. Under certain assumptions, we will classify such random homeomorphisms up to topological conjugacy.

Erdös-Rényi laws and local large deviations for unbounded observables on chaotic dynamical systems.

Matthew Nicol

Erdös-Renyi laws concern the almost sure behavior of time averages over time windows of varying length in a stationary time series. We discuss recent results on Erdös-Renyi laws for chaotic dynamical systems and their relation to rate of decay of correlations and local large deviations for unbounded observables. Some of this work is joint with Nicolai Haydn (University of Southern California), Holger Kantz and Mozhdeh Massah (Max Planck Institute for Complex Systems, Dresden, Germany).

Generic iterated function systems on the circle

Alexey Okunev

Joint work with V. Kleptsyn and Yu. Kudryashov.

An *iterated function system* (IFS) on a manifold M is a tuple of smooth maps $f_1, \ldots, f_s : M \to M$. One of the reasons for studying IFS's is that they (more precisely, associated step skew products over Bernoulli shift) provide a nice model example of partially hyperbolic skew products. If some interesting robust property is found for the IFS's, it is often possible to find this property for a locally generic set of diffeomorphisms (see, e.g., [1]).

Generic IFS's on the interval were studied by V. Kleptsyn and D. Volk ([2]). Among other things, they proved that the associated skew product has finitely many attractors and finitely many physical measures. However, it is unknown whether the supports of these physical measures coincide with the attractors.

Unlike IFS's on the interval, IFS's on the circle can be minimal (i.e., the $(f_1, \ldots, f_s)$-orbit of each point is dense). It turns out that this is the only difference from the interval case. Namely, for an open and dense set of IFS's on the circle such that $f_1, \ldots, f_s$ are orientation-preserving diffeomorphisms an alternative holds:

- either the IFS is minimal
- or there is an absorbing domain — a nontrivial finite union I of open intervals such that $f_i(I) \subset I$ for each $i = 1, \ldots, s$. In this case the results from [2] can be applied.

Though the result is purely topological, the proof is probabilistic. Let us equip each map f_i with a probability $p_i > 0$. Our key argument involves the stationary measure of this random dynamical system.

References

[1] Yu.S. Ilyashenko, *Thick attractors of boundary preserving diffeomorphisms,* Indagationes Mathematicae, 2011

[2] V. Kleptsyn, D. Volk, *Physical measures for nonlinear random walks on interval,* Mosc. Math. J., 2014

A Girsanov approach to slow parameterising manifolds in the presence of noise.

Christian Pangerl

We consider a three-dimensional slow-fast system with quadratic nonlinearity and additive noise. The associated deterministic system of this stochastic differential equation (SDE) exhibits a periodic orbit and a slow manifold. The deterministic slow manifold can be viewed as an approximate parameterisation of the fast variable of the SDE in terms of the slow variables. In other words the fast variable of the slow-fast system is approximately "slaved" to the slow variables via the slow manifold. We exploit this fact to obtain a two dimensional reduced model for the original stochastic system, which results in the Hopf-normal form with additive noise. Both, the original-, as well as the reduced system admit ergodic invariant measures describing their respective long-time behaviour.

We will show that there is a suitable metric on the space of probability measures on phase space such that the discrepancy between the marginals along the radial component of both invariant measures can be upper bounded by a quantity describing the quality of the parameterisation. However, we can show this only in a parameter regime, which is close to criticality. An important technical tool we use to arrive at this result is Girsanov's theorem, which allows us to modify the SDEs in question in a way that preserves the transition probabilities.

On the shortest distance between orbits and the longest common substring problem

Jerome Rousseau

We study the behaviour of the shortest distance between orbits and show that under some rapidly mixing conditions, the decay of the shortest distance depends on the correlation dimension. For irrational rotations, we prove a different behaviour depending on the irrational exponent of the angle of the rotation. For random processes, this problem corresponds to the longest common substring problem. Here, we extend the result of Arratia and Waterman on sequence matching to α-mixing processes with exponential decay. This is a joint work with Vanessa Barros and Lingmin Liao.

Quenched Decay of Correlations for Slowly mixing Systems

Marks Ruziboev

We study random Young towers with slowly decay tail of the return times and obtain upper bounds on the rates of decay of quenched correlations. We show that the random Liverani-Saussol-Vaienti maps with parameters in interval $[\alpha_0, \alpha_1]$, $0 \leq \alpha_0 < \alpha_1 < 1$ chosen independently with respect to distribution ν on $[\alpha_0, \alpha_1]$ admits random towers, and thus the quenched decay of correlation is governed by the fastest mixing map.

Stochastic bifurcation random logistic map

Yuzuru Sato

The emergence of noise-induced chaos in a random logistic map with bounded noise is understood as a two-step process consisting of a topological bifurcation agged by a zero-crossing point of the supremum of the dichotomy spectrum and a subsequent dynamical bifurcation to a strange random attractor agged by a zero-crossing point of the Lyapunov exponent. The associated three consecutive dynamical phases are characterized as a periodic random attractor, a point random attractor, and a strange random attractor, respectively. The rst phase has a negative dichotomy spectrum reecting uniform attraction to the periodic random attractor. The second phase no longer has a negative dichotomy spectrum. The xed point random attractor is not uniformly attractive, but it retains a negative Lyapunov exponent reecting the aggregate asymptotic con- tractive behaviour. For practical purposes, the extrema of the dichotomy spectrum equal that of the support of the spectrum of the nite-time Lyapunov exponents. We present detailed numerical results from various dynamical viewpoints, illustrating the dynamical characterisation of the three different phases

Almost sure limit variance in normal approximation for quenched random sequences of transformations

Mikko Stenlund

We study, in an abstract setting, the almost sure convergence rate towards normal distribution for random sequences of transformations. The talk focuses on the limit behavior of the variance of fiberwise centered sums and their convergence rates.

Periodic Random Dynamical Systems

Huaizhong Zhao

In this talk, I will introduce random periodic paths, periodic measures and their equivalence. I will present the ergodic theory in random periodic regime, the idea to Poincare sections (PS) and prove that PS-ergocity implies ergodicity. A necessary and sufficient conditions for PS-ergodicity is given in terns of the spectral structure of the Markov semigroup and their infinitesimal generator. An alternative necessary and sufficient condition is given in terms of the recurrence on the Poincare sections. The talk is based on the following two papers:

C.R. Feng and H.Z. Zhao, Random periodic processes, periodic measures and ergodicity, arXiv preprint arXiv:1408.1987.

C.R. Feng, B.Y.Qu and H.Z. Zhao, A Sufficient and Necessary Condition of PS-ergodicity of Periodic Measures and Generated Ergodic Upper Expectations, arXiv preprint arXiv:1806.03680.