

Roger Grimshaw (University College London, UK)

Title: Internal solitary wave generation by tidal flow over topography

Abstract: Internal solitary waves in the coastal ocean, straits and fjords are typically generated by barotropic tidal flow over localised topography. Wave generation can be characterised by the Froude number $F = U/c_0$, where U is the tidal flow amplitude and c_0 is the intrinsic linear long wave phase speed, that is the speed in the absence of the tidal current. For steady tidal flow in the resonant regime, $\Delta_m < F - 1 < \Delta_M$, a theory based on the forced Korteweg-de Vries equation shows that upstream and downstream propagating undular bores are produced. The bandwidth limits $\Delta_{\{m, M\}}$ depend on the height (or depth) of the topographic forcing term, which can be either positive or negative depending on the polarity of the topography relative to a solitary wave. The wave generation process is studied numerically using a forced Korteweg-de Vries equation model with time-dependent Froude number, $F(t)$, representative of realistic tidal flow. The response depends on $\Delta_{\{max\}} = F_{\{max\}} - 1$, where $F_{\{max\}}$ is the maximum of $F(t)$ over half of a tidal cycle. When $\Delta_{\{max\}} < \Delta_m$ the flow is always subcritical and internal solitary waves appear after release of the downstream disturbance. When $\Delta_m < \Delta_{\{max\}} < \Delta_M$ the flow reaches criticality at its peak, producing upstream and downstream undular bores that are released as the tide slackens. When $\Delta_{\{max\}} > \Delta_M$ the tidal flow goes through the resonant regime twice, producing undular bores with each passage. The numerical simulations are for both symmetrical topography, and for asymmetric topography representative of Stellwagen Bank and Knight Inlet where internal solitary waves are regularly observed.

Alexander Chesnokov (Novosibirsk State University, Russia)

Title: Internal Waves in a Two-Layer Stratified Liquid with Mixing and Shear

Abstract: Two-layer model of the vertical structure of ocean, where a layer of liquid of lower density is located over a layer of higher density, with a sharp interface, is quite realistic for coastal regions. Suppose that the river runoff occupies the upper shear layer of lower density, over the deep potential layer of higher density, and interaction with mixing occurs between them.

To study such flows, we propose a two-layer model where the upper layer is

considered within the framework of shear shallow water flows while the lower layer is almost potential and can be described by the Serre–Green–Naghdi or Saint-Venant type models. The interaction and mixing between the layers is taken into account through a kinematic boundary condition at the interface “shear layer – potential layer” describing an entrainment of potential fluid by a shear flow. The entrainment velocity is proportional to the intensity of large eddies in the upper layer. This approach was successfully applied in [1] to describe the spilling breakers in shallow water. In particular, the obtained model allows us to describe the transition from an undular bore to a breaking bore. A two-layer model for the interaction between a bubbly shear layer and long internal waves over topography was proposed in [2] where analytical and numerical results on the formation of large amplitude internal waves, including periodic and damped oscillating solutions were presented.

In this work we focus our attention on the internal waves in a two-layer stratified flow with shear and mixing. A modification of the governing equations is performed compared to [1, 2] which is specific to the modelling of stratified flows. We find a new type of solutions that represent a combination of periodic wave trains and table-top (or table-down) solitons.

[1] S.L. Gavriluk, V.Yu. Liapidevskii, and A.A. Chesnokov, *J. Fluid Mech.* 808, 441–468 (2016).

[2] S.L. Gavriluk, V.Yu. Liapidevskii, and A.A. Chesnokov, *Europ. J. Mech. B/Fluids*, (accepted).

John Grue (University of Oslo, Norway)

Title: Dead water: nonlinear and linear analysis, and comparison to observations.

Abstract: By strongly nonlinear and linear interfacial methods the dead water wake and drag due to realistic ship models in three dimensions are calculated at subcritical and supercritical speeds. The role of the nonlinearity is exemplified by comparison to observations. In the supercritical example, the lateral speed of the leading diverging wave, as observed in the field using images from camera, is substantially larger than the theoretical speed of the leading wave trough, but only slightly larger than the theoretical speed of the leading diverging wave slope. The speeds of waves number two and three in the field observation are very close to theory. In a second example

the strong increase in the dead water resistance at low subcritical speed ($Fr=0.5$) compares well to historical observations where the nonlinear theory captures the essential dynamics. Linear theory is found to be useless in the present examples.

Karima Khusnutdinova (Loughborough University, UK)

Title: Dynamics of interfacial waves on a shear flow

Abstract: In this talk I will overview some results for the dynamics of interfacial waves on a parallel shear flow. First, I will discuss the effects of the shear flow on interfacial waves in a rotating ocean [1,2]. We found first examples when the shear flow can change the sign of the rotation coefficient in the Ostrovsky equation, leading to unusual dynamics. I will also discuss the dynamics of two distinct linear long wave modes with nearly coincident phase speeds, described by the system of coupled Ostrovsky equations. Interestingly, the dominant features of the complex dynamical behaviour observed in numerical simulations can be classified and interpreted in terms of the main features of the linear dispersion curves, resembling the qualitative theory of ODEs. Finally, I will briefly discuss the effect of a parallel shear flow on the wavefronts of surface and interfacial ring waves [3,4].

References:

[1] A. Alias, R.H.J. Grimshaw, K.R. Khusnutdinova, On strongly interacting internal waves in a rotating ocean and coupled Ostrovsky equations, *Chaos* 23 (2013) 023121.

[2] A. Alias, R.H.J. Grimshaw, K.R. Khusnutdinova, Coupled Ostrovsky equations for internal waves in a shear flow, *Physics of Fluids* 26 (2014) 126603.

[3] K.R. Khusnutdinova, X. Zhang, Long ring waves in a stratified fluid over a shear flow, *Journal of Fluid Mechanics*, 794 (2016) 17-44.

[4] K.R. Khusnutdinova, X. Zhang, Nonlinear ring waves in a two-layer fluid, *Physica D*, 333 (2016) 208-221.

Wooyoung Choi (New Jersey Institute of Technology, USA)

Title: New asymptotic models for coupled surface and internal waves and their application to resonant wave interactions

Abstract: A Hamiltonian system describing the coupled dynamics of surface and internal waves in two-layer flows is obtained using asymptotic expansion. Then further reductions to simpler models are made to study resonant wave interactions. Numerical solutions of the system are discussed in comparison with some theoretical results of the reduced models.

Title: Spatial asymptotics for solitary waves in deep water

Miles Wheeler, Sam Walsh, and Ming Chen

Abstract: We consider the behavior near spatial infinity of localized traveling waves on the surface of an infinitely deep fluid. In a variety of settings and under suitable decay assumptions, we show that the leading order term in these asymptotics is of "dipole type". This has many implications for the wave, particularly in the simpler settings where the dipole moment in the expansion is given explicitly in terms of the kinetic energy. As an application, we provide detailed asymptotics for the waves with compactly supported vorticity constructed in [Shatah--Walsh--Zeng 2013].

Thierry Dauxois (ENS de Lyon & CNRS, France)

Title: Energy cascade in internal wave attractors

Abstract: Internal gravity waves play a primary role in geophysical fluids : they contribute significantly to mixing in the ocean and they redistribute energy and momentum in the middle atmosphere. In addition to their very interesting and very unusual theoretical properties, these waves are linked to one of the important questions in the dynamics of the oceans: the cascade of mechanical energy in the abyss and its contribution to mixing.

Combining the physics of waves, dynamical systems theory and oceanography, I will

discuss a unique self-consistent experimental and numerical setup that models a cascade of triadic interactions transferring energy from large-scale monochromatic input to multi-scale internal wave motion. I will also provide explicit evidence of a wave turbulence framework for internal waves

Ricardo Barros (Loughborough University, UK)

Title: Strongly nonlinear effects on mode-2 internal solitary waves

Abstract: Large amplitude internal waves in a three-layer flow confined between two rigid walls will be examined in this talk. The mathematical model under consideration arises as a particular case of the multi-layer model proposed by Choi (2000) and is an extension of the two-layer MCC (Miyata-Choi-Camassa) model. The model can be derived without imposing any smallness assumption on the wave amplitudes and is well-suited to describe internal waves within a strongly nonlinear regime. Emphasis will be given to the study of solitary-wave solutions, shown to be governed by a Hamiltonian system with two degrees of freedom. Solutions with one-hump profiles exhibited for different asymptotic limits. However, the richness of the dynamical system readily becomes apparent when certain regimes relevant to real oceanic applications and laboratory experiments are considered. In particular, we reveal the existence of multi-hump solutions in the case when stratification is weak and the density transition layer is thin.

Daniel Ratliff (Loughborough University)

Title: TBA

Title: Extended Lagrangian approach for the defocusing non-linear

Schrödinger equation

Sergey Gavriluk (Aix-Marseille Universite, France)

Abstract: Non-Linear Schrödinger (NLS) equation appears in the modelling for both surface and internal waves. We propose here a new method for the numerical resolution of the defocusing NLS equation written in hydrodynamic form by using the Madelung transform. From the mathematical point of view, hydrodynamical form can be seen as the Euler-Lagrange equations for a Lagrangian submitted to a differential constraint corresponding to the mass conservation law. The dispersive nature of NLS equation poses some major numerical challenges. The idea is to introduce a two-parameter family of extended Lagrangians, depending on a greater number of variables, whose Euler-Lagrange equations are hyperbolic and accurately approximate NLS equation in a certain limit. The corresponding hyperbolic equations are studied and solved numerically using Godunov type methods. Comparison of exact solutions to the NLS equation ('grey' solitons and dispersive shocks) and the corresponding numerical solutions to the extended system was performed. A very good accuracy of such a hyperbolic approximation was observed.

Title: Influence of surface gravity waves on submarine cables

Jeffrey Harris

Abstract: The recovery of offshore renewable energy, for example from tidal currents, requires submarine cables which may pass through regions with strong currents and waves, and as a result, it is necessary to understand the stability of such a cable, and therefore the forces involved. Most models of wave-current interactions with a slender cylinder are based on studies of offshore submarine pipelines, but given the different diameters and the typical seabed conditions in each application, not all aspects of the problem are identical. In particular, our application involves seabeds of high roughness, and therefore there is a significant interaction with the turbulent boundary layer. Considering established methods, early models of the problem based on a Morison approach have numerous limitations, and a Fourier decomposition approach is accurate but requires many coefficients, and does not provide much physical insight. Instead we consider instead a semi-empirical Wake model, which has been developed to include some physical understanding of the

problem, but which must be tuned to the flow regime considered. CFD modeling is used to calibrate the coefficients of the Wake model, and results will be presented comparing this approach with existing experimental data.

Paul Milewski (University of Bath, UK)

Title: TBA